The LAGRANGE points in the system Sun - Earth

Assumption: The center of gravity of the system sun - earth lies approximately in the center of the sun.
The earth moves on a circular orbit around the sun.
There are no other interferences.

given: $G := 6.6726 \cdot 10^{-11} \cdot \frac{N \cdot m^2}{kg^2}$

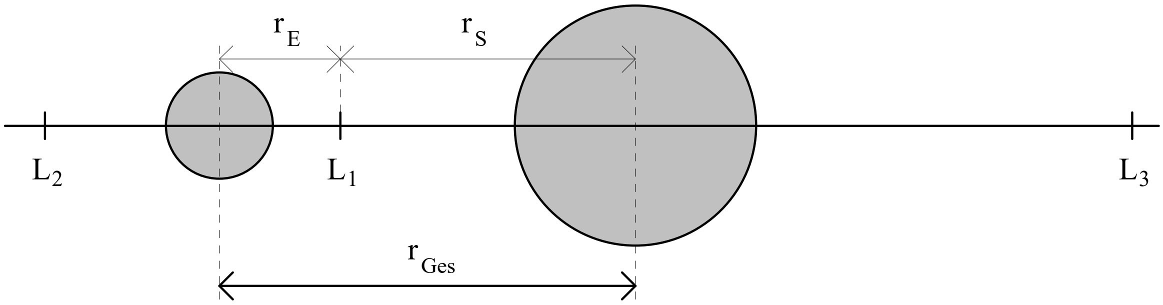
$m_S := 1.989 \cdot 10^{30} \cdot kg$; $r_{Ges} := 149.6 \cdot 10^{6} \cdot km$; $v_E := 30 \cdot \frac{km}{s}$

$m_E := 5.97 \cdot 10^{24} \cdot kg$; m_{Tel} ... mass of the telescope

$v_E := \sqrt{G \cdot \frac{m_S}{r_{Ges}}}$; $v_E = 29.785 \frac{km}{s}$

LAGRANGE point L_1:

Taking into account the movement of the earth follows (all bodies lie on one line):



$F_S = F_E + F_r$

$G \cdot \frac{m_S \cdot m_{Tel}}{r_S^2} = G \cdot \frac{m_E \cdot m_{Tel}}{(r_{Ges} - r_S)^2} + m_{Tel} \cdot \frac{v_{Tel}^2}{r_S}$; $T_E = \frac{2 \cdot \pi \cdot r_S}{v_{tel}} = \frac{2 \cdot \pi \cdot r_{Ges}}{v_E}$

$G \cdot \frac{m_S}{r_S^2} = G \cdot \frac{m_E}{(r_{Ges} - r_S)^2} + \frac{v_E^2 \cdot r_S}{r_{Ges}^2}$

$f(r) := G \cdot \frac{m_E}{(r_{Ges} - r)^2} + \frac{v_E^2 \cdot r}{r_{Ges}^2} - G \cdot \frac{m_S}{r^2}$

$\varepsilon_r := 100 \cdot km$

$$zero_1(lt, rt) := \left| \begin{array}{l} m \leftarrow \frac{lt + rt}{2} \\ m \ \text{ if } \ |rt - lt| < \varepsilon_r \\ \text{otherwise} \\ \quad \left| \begin{array}{l} zero_1(lt, m) \ \text{ if } \ (f(lt) \cdot f(m)) \le 0 \\ zero_1(m, rt) \ \text{ otherwise} \end{array} \right. \end{array} \right.$$

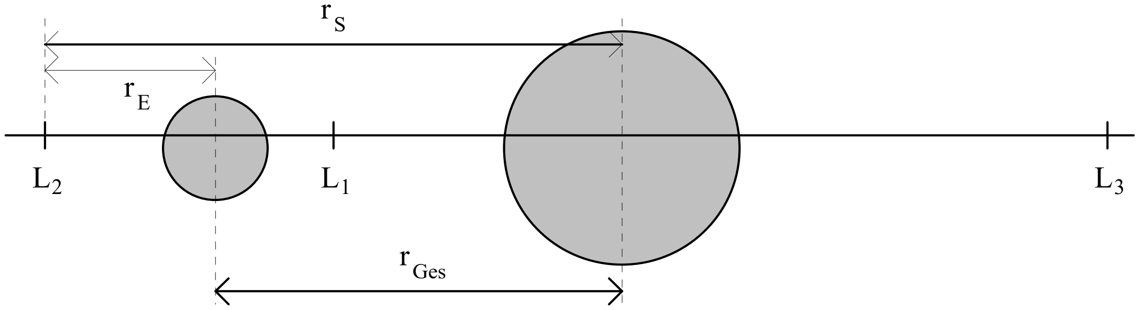
$r_S := zero_1(100000 \cdot km, r_{Ges})$

$r_S = 1.481 \times 10^{8}\,km$

$r_E := r_{Ges} - r_S$; $r_E = 1.491 \times 10^{6}\,km$

At a distance of approx. 1 491 000 km from the center of the earth, a body between the sun and earth is force-free, taking into account the earth's motion.

LAGRANGE point L_2:



$$F_r = F_E + F_S$$

$$m_{Tel} \cdot \frac{v_{Tel}^2}{r_S} = G \cdot \frac{m_E \cdot m_{Tel}}{(r_S - r_{Ges})^2} + G \cdot \frac{m_S \cdot m_{Tel}}{r_S^2} \quad ; \quad T_E = \frac{2 \cdot \pi \cdot r_S}{v_{Tel}} = \frac{2 \cdot \pi \cdot r_{Ges}}{v_E}$$

$$\frac{v_E^2 \cdot r_S}{r_{Ges}^2} = G \cdot \frac{m_E}{(r_S - r_{Ges})^2} + G \cdot \frac{m_S}{r_S^2}$$

$$f(r) := G \cdot \frac{m_E}{(r - r_{Ges})^2} + G \cdot \frac{m_S}{r^2} - \frac{v_E^2 \cdot r}{r_{Ges}^2}$$

$$\varepsilon_r := 100 \cdot km$$

$$zero_2(lt, rt) := \begin{array}{|l} m \leftarrow \frac{lt + rt}{2} \\ m \text{ if } |rt - lt| < \varepsilon_r \\ \text{otherwise} \\ \quad \begin{array}{|l} zero_2(lt, m) \text{ if } (f(lt) \cdot f(m)) \le 0 \\ zero_2(m, rt) \text{ otherwise} \end{array} \end{array}$$

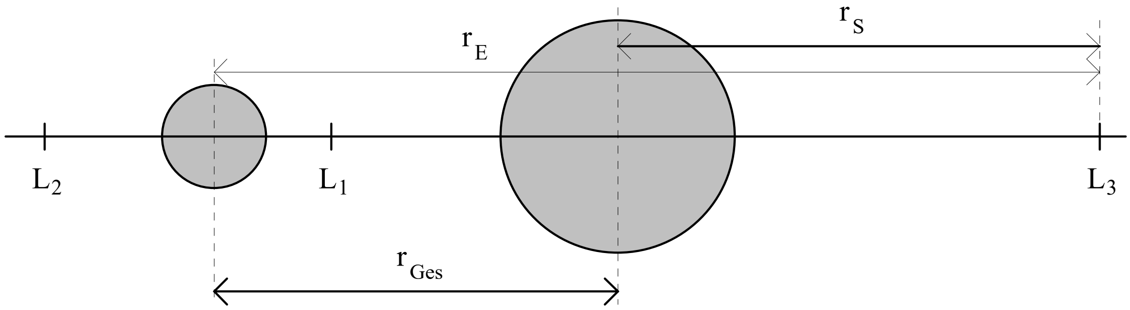
$$r_S := zero_2(r_{Ges} + 100000 \cdot km, 2 \cdot r_{Ges})$$

$$r_S = 1.511 \times 10^8 \, km$$

$$r_E := r_S - r_{Ges} \quad ; \quad r_E = 1.501 \times 10^6 \, km$$

The James Webb telescope must therefore be placed at a distance of about 1 500 000 km from Earth.

LAGRANGE point L_3:



$$F_r = F_E + F_S$$

$$m_{Tel} \cdot \frac{v_{Tel}^2}{r_E} = G \cdot \frac{m_E \cdot m_{Tel}}{(r_S + r_{Ges})^2} + G \cdot \frac{m_S \cdot m_{Tel}}{r_S^2} \quad ; \quad T_E = \frac{2 \cdot \pi \cdot r_S}{v_{Tel}} = \frac{2 \cdot \pi \cdot r_{Ges}}{v_E}$$

$$\frac{v_E^2 \cdot r_S}{r_{Ges}^2} = G \cdot \frac{m_E}{(r_S + r_{Ges})^2} + G \cdot \frac{m_S}{r_S^2}$$

$$f(r) := G \cdot \frac{m_E}{(r + r_{Ges})^2} + G \cdot \frac{m_S}{r^2} - \frac{v_E^2 \cdot r}{r_{Ges}^2}$$

$$\varepsilon_r := 100 \cdot km$$

$$zero_3(lt, rt) := \left| \begin{array}{l} m \leftarrow \frac{lt + rt}{2} \\ m \ \text{ if } \ |rt - lt| < \varepsilon_r \\ \text{otherwise} \\ \quad \left| \begin{array}{l} zero_3(lt, m) \ \text{ if } \ (f(lt) \cdot f(m)) \le 0 \\ zero_3(m, rt) \ \text{ otherwise} \end{array} \right. \end{array} \right.$$

$$r_S := zero_3(1000000 \cdot km, r_{Ges})$$

$$r_S = 1.496 \times 10^8 \, km$$

$$r_E := r_S + r_{Ges} \quad ; \quad r_E = 2.992 \times 10^8 \, km$$

Hint: There are two more LAGRANGE points, which move outside of the connecting line Sun - Earth on the elliptical Earth orbit.