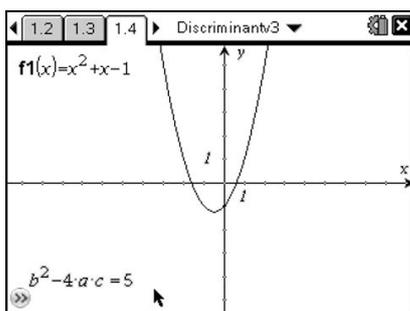


## The Discriminant – Introduction

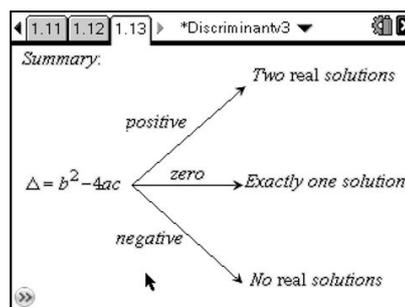
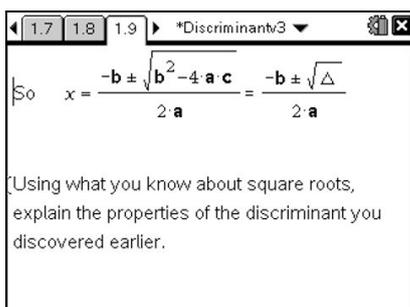
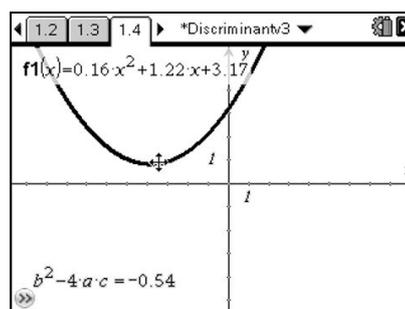
<b>Mathematical Content:</b> Quadratic Functions Discriminant	<b>Technical TI-Nspire Skills:</b> Manipulation of Graphs Translation and Dilating Graphs Questions & Answers
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One of the key elements in the classification of quadratic equations is identifying the number of roots the functions has. This is frequently done by looking at the value of the discriminant. In this activity students can explore the value of the discriminant in an experimental way by observing the number of roots and the value of the discriminant whilst manipulating the quadratic function. Having explored the problem experimentally, students are then encouraged to look at why the discriminant should have this property by looking at its place within the quadratic formula.



This activity could either be used by copying it to students' individual handhelds or could be used by the teacher using the TI-Nspire software to explore the topic with the whole class.

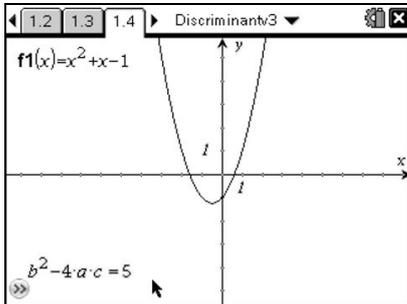
During the task students explore the discriminant experimentally by manipulating a quadratic graph and observing the effect this has on the discriminant. They are then asked to work through a series of questions that encourage them to summarise their explorations. There is also a series of notes pages that explore the discriminant in the context of the quadratic equation.



## The Discriminant – Student Worksheet

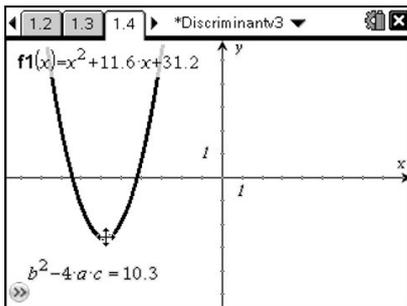
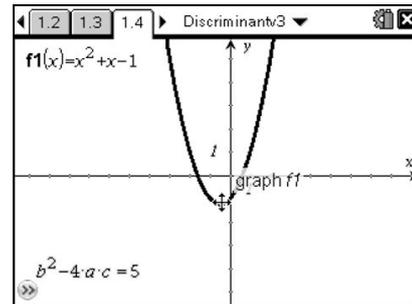
*In this task you will explore the value taken by the discriminant for quadratic equations in various different scenarios.*

You should begin this task by opening the Discriminant.tns file. The first few pages introduce the task and you can navigate between these by pressing  $\text{ctrl}$   $\leftarrow$   $\rightarrow$



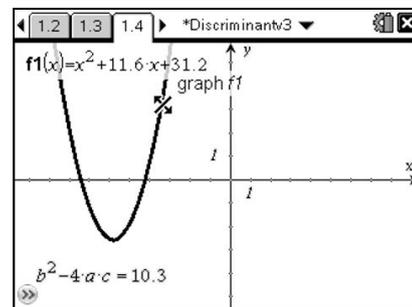
On Page 1.4 you will be presented with a quadratic graph. You can see the function at the top left of the page and the value of the discriminant at the bottom left of the screen.

To translate the graph you can drag it by clicking on the vertex of the graph: either press  $\text{ctrl}$   $\leftarrow$   $\rightarrow$  or press and hold  $\leftarrow$   $\rightarrow$ .



**Note:** that the cursor changes to become a cross hair indicating that you can translate the graph. To stop translating the graph either press  $\text{esc}$  or press  $\leftarrow$   $\rightarrow$  again.

You can stretch the graph along the x-axis by moving the cursor further up the curve and clicking and dragging. Notice that the cursor changes to  $\leftarrow$   $\rightarrow$  signifying that it is ready to stretch the graph.



Having spent some time experimenting with translating and stretching the graph, you should be able to form a hypothesis about the relationship between the graph and the value of the discriminant.

When you have some thoughts about the relationship between the graph and the value of the discriminant, have a look at pages 1.5, 1.6 and 1.7, which ask 3 specific questions about this relationship. Press **(tab)** and type your answer.

1.5 1.6 1.7 \*Discriminant3

For what values of  $\Delta = b^2 - 4 \cdot a \cdot c$  are there two distinct roots?

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1.5 1.6 1.7 \*Discriminant3

For what values of  $\Delta = b^2 - 4 \cdot a \cdot c$  are there no roots?

---



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1.5 1.6 1.7 \*Discriminant3

For what value of  $\Delta = b^2 - 4 \cdot a \cdot c$  is there exactly one root?

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Once you are satisfied with your answer you can check it by pressing **(menu)** and going to "Check Answer".

On pages 1.8 and 1.9 you are reminded of the quadratic formula and its relationship to the discriminant.

1.6 1.7 1.8 \*Discriminant3

Hopefully you are now asking yourself why this should be the case.

You may recall the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

Look carefully and you should notice the discriminant in there.

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1.7 1.8 1.9 \*Discriminant3

So  $x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{-b \pm \sqrt{\Delta}}{2 \cdot a}$

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Using what you know about square roots, explain the properties of the discriminant you discovered earlier.

On page 1.10 you are asked to explain, with reference to the properties of square roots and the quadratic formula, why the results you found earlier must be true.

1.8 1.9 1.10 \*Discriminant3

Why must there be two roots when the discriminant is positive?

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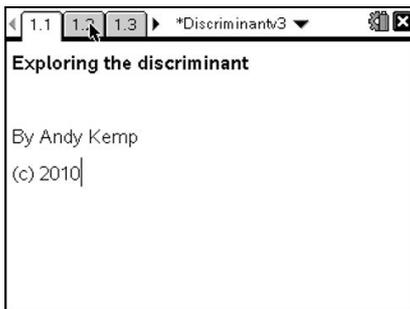
Again when you are confident with your answer you can check it by pressing **(menu)** **(2)**.

The final page 1.13 acts as a summary of the results you have found and you may like to copy it in to your notes to help you remember the various cases.

## The Discriminant – Detailed Notes for Teachers

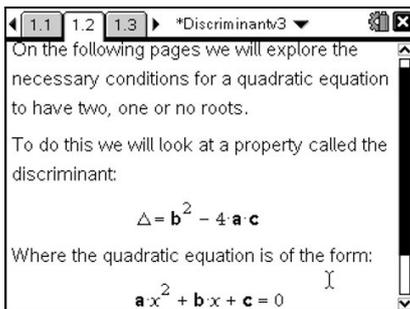
These notes briefly describe the content of each page and draw attention to any important elements.

Page  
1.1



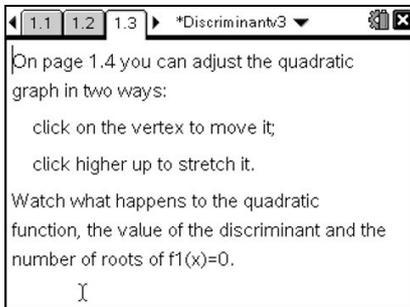
This first page acts as a title page.

Page  
1.2



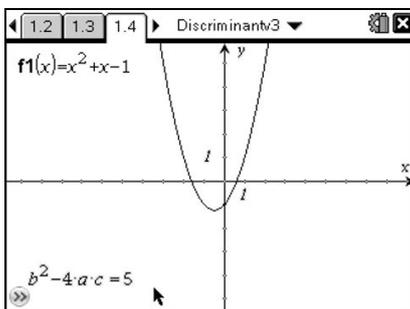
This page explains what the discriminant is.

Page  
1.3



The third page tells the student how to manipulate the graph by either translating or stretching.

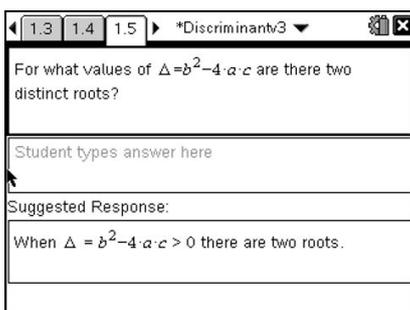
Page  
1.4



On this page students can manipulate the graph by dragging either the vertex to translate the graph or a point further up the graph to stretch it horizontally.

Students should observe the effect these transformations have on the discriminant and on the number of roots of  $f_1(x)=0$ .

Page  
1.5



On this page students are asked to describe the value of the discriminant when there are two distinct roots.

Students should respond that there are two distinct roots when the discriminant is positive.

Page 1.6

For what values of  $\Delta = b^2 - 4ac$  are there no roots?

Student types answer here

Suggested Response:

When  $\Delta = b^2 - 4ac < 0$  there are no roots.

On this page students are asked to describe the value of the discriminant when there no real roots.

Students should respond that there are no roots when the discriminant is negative.

Page 1.7

For what value of  $\Delta = b^2 - 4ac$  is there exactly one root?

Student types answer here

Suggested Response:

When  $\Delta = b^2 - 4ac = 0$  there is one root.

On this page students are asked to describe the value of the discriminant when there is exactly one root.

Students should respond that there is exactly one root when the discriminant is zero.

1: Clear Answers  
2: Check Answer  
3: Insert  
4: Format  
5: Teacher Tool Palette  
6: Hints

When it is zero

For all of these Q&A pages students can check their answers at any point by pressing (menu) (2) to select "Check Answer".

Page 1.8

Hopefully you are now asking yourself why this should be the case.

You may recall the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Look carefully and you should notice the discriminant in there.

On page 1.8 students are reminded about the quadratic formula and are asked to consider its relationship to the discriminant.

Page 1.9

So  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{\Delta}}{2a}$

Using what you know about square roots, explain the properties of the discriminant you discovered earlier.

Here the links between the quadratic formula and the discriminant are made much more explicit and students are asked to consider, using what they know about square roots, how this justifies what they previously discovered about the discriminant.

Page 1.10

Suggested Response:

As the roots are given by  $x = \frac{-b \pm \sqrt{\Delta}}{2a}$ ,  
if  $\Delta > 0$  then it has two real square roots and therefore the two solutions are  
 $x = \frac{-b + \sqrt{\Delta}}{2a}$  and  $x = \frac{-b - \sqrt{\Delta}}{2a}$ .

On this page the student is asked to explain "Why must there be two distinct roots when the discriminant is positive?"

Page 1.11

Why must there be exactly one root when the discriminant is zero?

Student types answer here

Suggested Response:

As the roots are given by  $x = \frac{-b \pm \sqrt{\Delta}}{2 \cdot a}$ , if  $\Delta = 0$  then  $\sqrt{\Delta} = 0$  and therefore the only solution is  $-b$

On this page the student is asked to explain “Why must there be exactly one root when the discriminant is zero?”

Page 1.12

Why must there be no real roots when the discriminant is negative?

Student types answer here

Suggested Response:

As the roots are given by  $x = \frac{-b \pm \sqrt{\Delta}}{2 \cdot a}$ , if  $\Delta < 0$  then  $\sqrt{\Delta}$  is not real because there is no real square root of a negative number. Therefore there

On this page the student is asked to explain “Why must there be no real roots when the discriminant is negative?”

Again on all these pages students can check their answers by pressing the  button and selecting “Check Answer”.

Page 1.13

Summary:

$\Delta = b^2 - 4ac$

- positive → Two real solutions
- zero → Exactly one solution
- negative → No real solutions

This final page is a summary of what the student has discovered. Students may wish to copy this diagram into their notes for future reference.