Perfect pitch Introduction

This activity looks at connections and patterns in the frequencies and wavelengths of musical notes, and links mathematics to music and science. Using TI-Nspire can make both the mathematics and the science easier to understand.

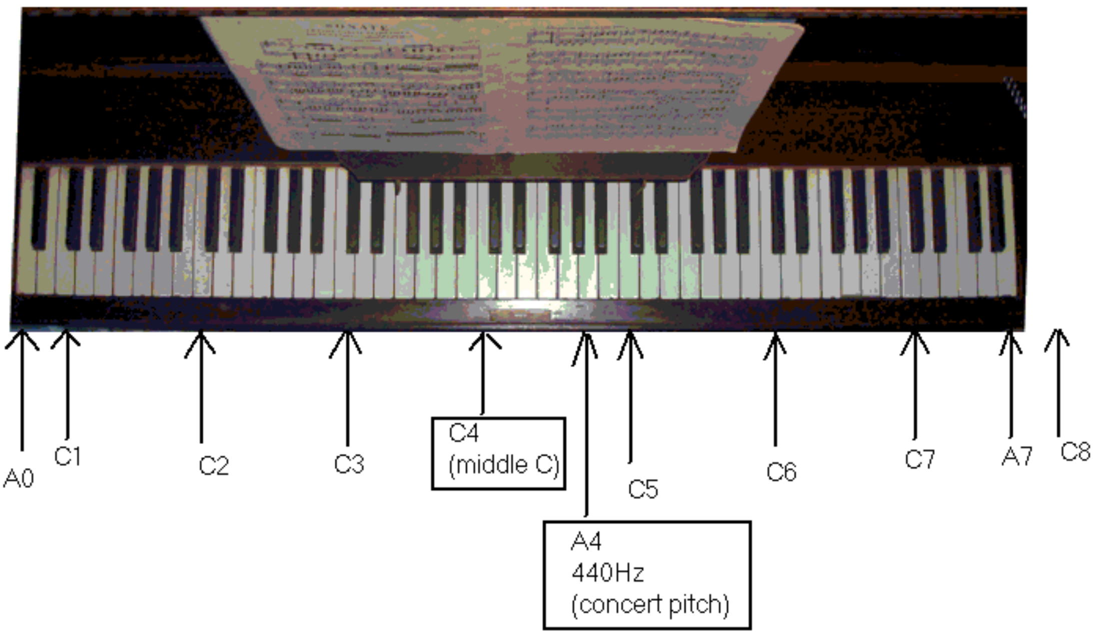
Background information

Have you ever thought about how musical notes are produced? What happens when you pluck a guitar string or press a key on a keyboard? What makes high or low notes?
Vibrations of strings or columns of air produce sound waves. Sound waves are longitudinal waves. The vibration is in the direction that the wave travels. The number of complete vibrations or cycles per second that produce a particular note is called the frequency and is measured in Hertz (Hz).

In recent centuries the frequencies of notes played by different instruments of an orchestra have been standardised using an 'equal tempered scale' so that they sound in tune with each other despite being in different keys.
The instruments of an orchestra all tune up before playing to a particular note – the A above middle C on a piano often referred to as A4 with a standardised frequency of 440Hz. This is shown in the diagram below.

Different systems of labelling notes can be found but just two systems will be referred to in this activity.

1. **Using conventional names for notes**
   One system is to start at C0 which is below the usual lowest note on a full-sized piano keyboard and each octave after that will be C1, C2, C3, etc as shown in the image below.



   Between the white notes shown on the keyboard the black notes are either flats or sharps. They are named after the adjacent white notes. If they are slightly below the note then they are flats (b) and slightly above then they are sharps (#). For example the black key between an A and a B could be labelled either $A^{\#}$ or B^{b}

2. **The MIDI system**

The MIDI system for electronic musical instruments and computers uses a straight count starting with note 0 for C-1 at 8.1758 Hz up to note 127 for G9 at 12,544 Hz.

C-1 and G9 are outside the range of the piano keyboard shown. C-1 is one octave below C0 and 2 octaves below C1 which is the first C to be shown on the keyboard above. At the other end G9 is almost 2 octaves above the highest note shown.

The table left shows some examples of notes that are on the piano keyboard using both labelling systems.

21	A_0
22	$A^{\#}_0/B^{b}_0$
23	B_0
24	C_1
25	$C^{\#}_1/D^{b}_1$
26	D_1
27	$D^{\#}_1/E^{b}_1$
28	E_1
29	F_1
30	$F^{\#}_1/G^{b}_1$

The activity

At the end of this activity is an extract from a set of data relating note numbers, names, frequencies and wavelengths.

- Do you notice any patterns or possible relationships in the table?
- Can you spot any patterns in the note frequencies?
- What sort of relationship do you think there is between the frequency and the wavelength?
- How can you use the facilities of Ti-Nspire to help you with this?

Some ideas and hints to get started

Here are two possibilities that you could think about investigating.

A Note frequencies: Looking for sequences, patterns, predictions and rules

1. You could look first at the A notes since A4 is the one used for orchestral tuning.
   - What do you notice about the frequencies of the A notes starting with A1?
   - Can you find a rule to get from the frequency of one A note to the frequency of the one above?
   - Can you find a rule connecting the note number to the frequency?

   There is a tns files with this information partly set up to help you.
   You could try out your rule by inserting a 'data and statistics' page and graphing 'anote' (the A note number) on the x-axis against 'afreq' (the corresponding frequency) on the y-axis. Use 'plot function' to try out your rules.
2. You could then go on to look at sequences of other notes such as C and so on. Can you find any general patterns or rules?
3. Can you find a general rule to match all the data in the table? You could try using a 'Data and Statistics 'page and graph the note numbers against their frequency. Then try different types of regression on menu 4 to see if you can find a function that fits the graph.
4. An interval of one note is called a semi-tone; two notes would be a tone for example from C1 to D1. The Pythagoreans (followers of Pythagoras) were intrigued by musical harmony and tried to describe it mathematically. They reckoned that to raise a note by one tone you needed to increase the frequency by multiplying by $\frac{9}{8}$. How well does this rule fit the data in the table?

B Frequency and wavelength: What is the relationship?

- Look at the data. What happens to the wavelength as the frequency increases?
- What sort of relationship is this? Why?
- What would a graph of this relationship look like? Can you find an equation?
- What do you think wavelength means? How could you work it out if you knew the frequency of the note?

Hints:

1. You could try inserting an extra column in the spreadsheet and putting in a formula to multiply the frequency by the wavelength. What do you notice? How could this information help you to find an equation for the graph?
2. You could try using a 'Data and Statistics 'page and 'Plot function' on menu 4 to help you. What do you notice?
3. The speed that sound travels is approximately 344 metres per second. How is this related to frequency and wavelength?

Further information from:-

http://www.phy.mtu.edu/~suits/notefreqs.html
http://en.wikipedia.org/wiki/Equal_temperament
http://en.wikipedia.org/wiki/MIDI
http://www.phy.mtu.edu/~suits/NoteFreqCalcs.html

Midi note number	Note	Frequency (Hz)	Wavelength (cm)		Midi note number	Note	Frequency (Hz)	Wavelength (cm)
12	C_0	16.35	2100		**45**	A_2	110	314
13	$C^{\#}_0/D^b_0$	17.32	1990		**46**	$A^{\#}_2/B^b_2$	116.54	296
14	D_0	18.35	1870		**47**	B_2	123.47	279
15	$D^{\#}_0/E^b_0$	19.45	1770		**48**	C_3	130.81	264
16	E_0	20.6	1670		**49**	$C^{\#}_3/D^b_3$	138.59	249
17	F_0	21.83	1580		**50**	D_3	146.83	235
18	$F^{\#}_0/G^b_0$	23.12	1490		**51**	$D^{\#}_3/E^b_3$	155.56	222
19	G_0	24.5	1400		**52**	E_3	164.81	209
20	$G^{\#}_0/A^b_0$	25.96	1320		**53**	F_3	174.61	198
21	A_0	27.5	1250		**54**	$F^{\#}_3/G^b_3$	185	186
22	$A^{\#}_0/B^b_0$	29.14	1180		**55**	G_3	196	176
23	B_0	30.87	1110		**56**	$G^{\#}_3/A^b_3$	207.65	166
24	C_1	32.7	1050		**57**	A_3	220	157
25	$C^{\#}_1/D^b_1$	34.65	996		**58**	$A^{\#}_3/B^b_3$	233.08	148
26	D_1	36.71	940		**59**	B_3	246.94	140
27	$D^{\#}_1/E^b_1$	38.89	887		**60**	C_4	261.63	132
28	E_1	41.2	837		**61**	$C^{\#}_4/D^b_4$	277.18	124
29	F_1	43.65	790		**62**	D_4	293.66	117
30	$F^{\#}_1/G^b_1$	46.25	746		**63**	$D^{\#}_4/E^b_4$	311.13	111
31	G_1	49	704		**64**	E_4	329.63	105
32	$G^{\#}_1/A^b_1$	51.91	665		**65**	F_4	349.23	98.8
33	A_1	55	627		**66**	$F^{\#}_4/G^b_4$	369.99	93.2
34	$A^{\#}_1/B^b_1$	58.27	592		**67**	G_4	392	88
35	B_1	61.74	559		**68**	$G^{\#}_4/A^b_4$	415.3	83.1
36	C_2	65.41	527		**69**	A_4	440	78.4
37	$C^{\#}_2/D^b_2$	69.3	498		**70**	$A^{\#}_4/B^b_4$	466.16	74
38	D_2	73.42	470		**71**	B_4	493.88	69.9
39	$D^{\#}_2/E^b_2$	77.78	444		**72**	C_5	523.25	65.9
40	E_2	82.41	419		**73**	$C^{\#}_5/D^b_5$	554.37	62.2
41	F_2	87.31	395		**74**	D_5	587.33	58.7
42	$F^{\#}_2/G^b_2$	92.5	373		**75**	$D^{\#}_5/E^b_5$	622.25	55.4
43	G_2	98	352		**76**	E_5	659.26	52.3
44	$G^{\#}_2/A^b_2$	103.83	332					